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**DIDACTIC CONTRACTS IN REALISTIC MATHEMATICS EDUCATION
TEACHING PRACTICE IN INDONESIA:
A LESSON ON ADDITION**

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Abstract

This paper aims to investigate characterize features of didactic contracts in realistic mathematics education teaching practice in Indonesia in the case of a lesson on addition. We just focus on some episodes of 26 first grade students and a female teacher from SDN 197 Palembang learning combinations that make ten based on a Palembang traditional food, pempek, and tablets of medicine. The result shows that some features such as formulation and validation appear during the teaching and learning process. The students are able to produce combinations that make ten individually and collectively, and also make agreement for those combinations. The teacher has an important role to guide students to the process of institutionalized knowledge.

Keywords: didactic contracts, combinations that make ten, realistic mathematics education

INTRODUCTION

Research on mathematics education is a broader area in which one of them aimed to improving mathematics teaching and learning practices. There are two grand types of approaches to improving mathematics teaching and learning practices (Miyakawa & Winsløw, 2009). The first one is an ordinary teaching and learning practice in which researchers design a lesson based on analysis of mathematical tasks that teacher has experience in the real classroom. The second one is a specific model design of teaching and learning practice in which a lesson is designed by researchers based on a theoretical principle proposed by a theory.

One of the theories used to analyse mathematical teaching and learning practices is the theory of didactical situations (TDS; cf. Brousseau, 1997) with a specific situation related to didactic contracts. Even in many articles, the TDS was used to analyse the ordinary teaching and learning practices, it can be challenged when we use it to analyse a particular design of teaching and learning practice such as a realistic mathematics education (RME) teaching practice. In this paper, we study a lesson of addition in a first grade class in Indonesia, based on principles of RME, and try to answer the following question.

1. What are the characteristic features of the didactic contracts in key phases of the lesson, such as formulation and validation situations?
2. Can these features be related to the design format of RME? How?

This paper structures into some sections as follows: First, we provide a short theoretical framework related to RME as a design format, didactic contracts, and combinations that make ten. Then, we describe methodological approaches in collecting, designing, and analysing the data. After that, we analyse the classroom situation using the TDS specifically the didactic contracts. Finally, we discuss the result and address the research questions explicitly.

LITERATURE REVIEW

Realistic Mathematics Education (RME)

Realistic Mathematics Education (RME) is a domain-specific instruction theory of mathematics, which firstly has been developed in the Netherlands (Van den Heuvel-Panhuizen & Drijvers, 2014), and then it was adopted in Indonesia since 2001. The term of realistic is not always a real-world situation, but it can be a situation in which students can imagine even it is a formal word of mathematics, as far as it is real in their mind. The idea of RME itself comes from Freudenthal (1983) that is a *didactical phenomenology*. It is about how to teach mathematics,

how students can be brought to higher level of understanding of mathematics concepts, even the term of understanding is really hard to define.

According to Feudenthal (1991), mathematics is a human activity so mathematics should be learnt as an activity of mathematizing reality or mathematics. Mathematizing process or mathematization can be differentiated into horizontal and vertical forms (Treffers, 1987; Feudenthal, 1991). In *horizontal mathematization*, a real world problem is transformed into mathematical symbols. Meanwhile, the process of moving the abstract world of symbols or connections between concepts and strategies is called *vertical mathematization*.

RME has six core principles: an *activity principle*, a *reality principle*, a *level principle*, an *intertwinement principle*, an *interactivity principle*, and a *guidance principle* (Van den Heuvel-Panhuizen & Drijvers, 2014). Those principles can be used in designing a teaching and learning practice, for instance: a lesson of addition. The *activity principle* mainly concerns with students are active in the learning process. The *reality principle* is about a problem or task given to students should be meaningful and contextual situations. The *level principle* is that the process of learning mathematics starting from model of a particular situation to model for general situation. Then, the *intertwinement principle* is about relationship among some domains such as addition and subtraction. The *interactivity principle* mainly focuses on social interactions between a teacher and students and among students in a classroom discussion. Finally, the *guidance principle* is that a teacher has a proactive role in students' learning. Those principles are used in this research to design the learning process of first grade students in learning addition.

Didactic Contract (DC)

The concept of *didactic contract* was introduced to mathematics education by Brousseau (1997) to analyse a classroom interaction. The *didactic contract* is a way to negotiate and share responsibilities and expectation of a teacher and students with respect to mathematical knowledge at stake (Hersant & Perrin-Glorian, 2005). There are two roles of the teacher in the *didactic contract* namely *devolution* and *institutionalization*. The *devolution* is a process when the teacher gives a responsible to the students to obtain new mathematical knowledge by solving a problem or giving an appropriate situation from a certain milieu. Then, the next step is the *institutionalization* process that the students relate between their previous and new knowledge to solve other problems in a new situation.

There are four dimensions of a *didactic contract* (Hersant & Perrin-Glorian, 2005). These are the mathematical field or domain, the didactical status of the knowledge, the nature and characteristics of the ongoing *didactic situation*, and the distribution of the responsibility with respect to the knowledge at stake. Those four dimensions are interconnected. For instance, in the case of learning addition up to 10 as a domain, there is previous knowledge of students such as knowing numbers up to 10 as the didactical status of the knowledge. Then, the process moves to *institutionalization* of knowledge that is a part of ongoing didactic situation. Finally, the teacher shares the responsibility to gain a new knowledge such finding an addition of two numbers up to 10.

The structure of the *didactic contract* is distinguished into three levels: the *macro-*, the *meso-*, and the *micro-contract* (Hersant & Perrin-Glorian, 2005). In this research, we focus on the awareness of the lesson of addition, the level of the *meso-contract*, and an episode focused on mathematical moments, the level of the *micro-contract*.

Addition: Combinations that make ten

Based on a competence standard of National Education Curriculum 2006, First grade students should be able to add and subtract number up to 20 in the first semester and up to 100 in the second semester (Badan Standar Nasional Pendidikan, 2006). There is a link between students' knowledge in the first semester to the second semester. The goal is that they are able to use their knowledge on number facts up to 20 to solve addition and subtraction up to 100 with flexible strategies.

In many cases, students use counting one by one as a basic strategy to solve addition problems (Sarama & Clements, 2009). It will be useful when they solve addition problems with small numbers, but they probably will find some difficulties when they work with numbers above 20, so they have to use more abbreviated strategies. One of important mathematical ideas is combinations that make ten because it is a basis of other facts (Fosnot & Dolk, 2001). This idea is useful to solve addition problems by making ten and then adding ones. For example, to solve problem like $8 + 4$ by making a ten is $8 + 2$ and adding 2. Otherwise they tend to use counting on strategy.

METHOD

For this paper, the data were from an experimental teaching and learning practice in 2011 based on realistic mathematics education (RME) approach. The subject was a female teacher and 26 first grade students from SDN 179 Palembang, Indonesia. We observed some episodes of two weeks teaching and learning practices about addition up to 20, and focused our observation and analysis on combinations that make ten were done. The target knowledge for this teaching and learning activity is that students are able to find (x,y) that fulfil $x + y = 10$. In order to find those combinations, it was provided some artefacts: colourful markers, cartons, two kinds of *pempeks*, Palembang traditional foods, plates, and tablets of medicine used for another activity.

The data were gained from students' worksheets and videos. Some interaction between the teacher and students or among students especially in the phases of the didactic contracts appear and divide into some episodes were transcribed. Then, the data are analysed based on the characteristic features of the *didactic contracts*. The investigation of the *formulation* and *validation* situations occurred during the teaching and learning practice, and characterize mathematical moments in the level of *meso-* and *micro- contract*. Finally, the examination of those features relate to the six core principles of RME was done.

RESULTS

The result started by presenting an overview of the lesson from the video data and students' worksheets. The goal of the lesson is to find combinations that make ten. The lesson is presented in the context of a Palembang traditional food, *pempek*. All students know about this situation, and it will help them to participate actively in the lesson. The real *pempek* wasn't used in the teaching and learning practice, but it was changed into the imitation *pempek* made from wax. Actually there are several kinds of *pempek*, but two kinds of them: the *pempek dos* (look like a ball) and the *pempek lenjer* (look like a tube) was chosen.

A priori analysis for this activity is that students will start make a combination of ten by 5 and 5 since it can be recognized from fingers or mental arithmetic. The next hypothesis is that they add 1 to 5 to get 6 and subtract 1 from 5 to get 4, so the new combination that makes ten is 6 and 4. The other hypothesis is that they probably first think a number such as 3 then find what the other number to make 10 ($3 + . . . = 10$). In more detail what happens during the teaching and learning process can be seen on the episodes describing bellows:

Episode 1: The teacher started the class by showing a *pempek* picture, and all students directly recognize it. After a short conversation between the teacher and students about their experiences with the *pempeks*, she showed a plastic bag of *pempeks* (actually the imitation *pempeks*), and asked a student, Fadil, to put 10 mixed *pempeks* into a plate. While he put *pempek* into a plate one by one, other students followed it by saying number 1 to 10. He made a combination of ten by taking 5 *pempek doses* and 5 *pempek lenjers*. To bring this idea to the whole class, the teacher asked other students about the arrangement of *pempeks* by Fadil.

Teacher	: How many all of these? (while holding the plate)
A student	: 10. (weak voice)
Some students	: 5. (loud voice)
Teacher	: How many in total? (move from sitting to standing)
Students	: 10.

Teacher : How many *pempek lenjers*?
Students : 5.
Teacher : How about *pempek doses*?
Students : 5.
Teacher : So, how many in totals?
Students : 10.
Teacher : Please, check it Fadil.

Then Fadil checked his answer and agreed with it.

The *interactivity* occurring between the teacher and students took them to the process of formulation and validation (*micro-contract* of agreement). They formulated a combination, 5 and 5, to make a ten. In the end, the teacher again asked the students to validate their answers, and Fadil also validated his answer that is true 5 and 5 is 10 (*micro contract* of agreement).

The teacher then asked the other student, Rizki, to arrange 10 *pempeks* in the same manner but difference arrangement. By doing this, she made Rizki and other students to think other combinations based on their old knowledge: for each value of x , there is only one value of y , so $x + y = 10$. It could be seen from interactions among the teacher, Aditya and other students.

Teacher : Rizki, please take *pempeks* but do not take 5 and 5, another one!
Then, Aditya interrupts it.
Aditya : How many?
Some students : 4 and 6. (weak voice)
Teacher : Basically, the totals must be 10.
Aditya : 5 and 5 to make 10.
Teacher : Do not 5 and 5.
Aditya : 5 and 10.
Some students : 7 and 3. (Speak weakly and unclearly)

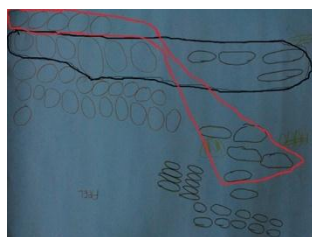
In this interaction, it seems that Adytia just though that only 5 and 5 to make 10, meanwhile some students could figure out certain combinations such as (4, 6), and (7, 3). The teacher did not validate those answers instead of waiting for Rizki's arrangement. Rizki arranged 6 *pempek lenjers* and 4 *pempek doses*. After she asked him to validate his answer, she showed this combination by her fingers to the other students. She thus emphasized the *institutionalization* of knowledge in progress.

However, a *mathematical conflict* occurred when the teacher asked the students why combinations of ten providing by two students, 5 and 5 by Fadil and 6 and 4 by Rizki, give the same result that was 10. We observed that no students answered this question because they got difficulty to justify and explain why the result always 10. Then, she asked the students what number were added for Rizki's arrangement. Aditya and another student answered 4 plus 6, and then Aditya also said 6 plus 4. Here, he had an idea about the number relationship and move from concrete to abstract mathematics. After that, she also asked again a combination arranged by Fadil, and highlights the students to realize that two combinations could make the same result that is 10.

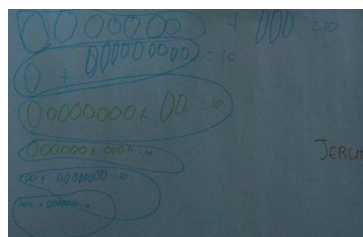
Episode 2: The students worked in groups by 4 or 5 students, so there were 6 groups in totals. The name of each group was based on the name of fruits: durian, mango, apple, papaya, banana, and orange. The teacher just gave an instruction that the students worked in groups to find as many combinations as possible that make ten. She gave each group a plastic bag of *pempeks* containing more than 10 for each kind of *pempeks*. We observed that apple, orange, and mango groups, used *pempeks* to find a combination that makes ten. Two other groups, papaya and banana, used fingers, and the other group recognized combinations that make ten mentally (figure 1). They had different approaches to formulate their answers. Even if the teacher provided artefacts, *pempeks*, it was not really useful for some students who looked at an opportunity to use fingers as a tool and know number facts mentally.

The apple group presented an unstructured drawing, so it was quite difficult to find combinations they make from their drawing. Meanwhile, we could recognize other group works easily since they presented their answer in good structures. Four groups wrote arithmetic operations (+ and =), and the durian group wrote numerical symbols and then pictorial representations. This group institutionalized the contextual situation into the mathematical symbols. However, the pictorial representations were drawn by students after the teacher's intervention. The role of the teacher was too strong and could stop the process of breaking the didactic contracts.

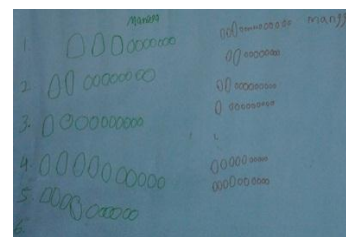
The roles of students finding combinations that make ten were varied. For instance, the banana group, they first took two *pempek doses* and put on the plate, and thought how many *pempek lenjers* were needed to make ten. Then they drew it on their poster. Mathematically, we can write as $2 + y = 10$, and find y by counting on or know mentally. Meanwhile, when we observed the orange group making a combination that make ten (8 *pempek lenjers* and 2 *pempek doses*), They first took 2 *pempek doses* and then took by 2 *pempek lenjers* until they got 10 in totals. Among those groups, we observed that only the durian group directly wrote combinations that make ten mentally. It means that they used their knowledge about number facts that make ten such as $1+9=10$, and then found other combinations from it. The process of *institutionalization* occurring during working in groups were varies among students.



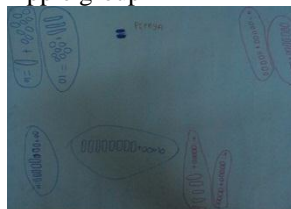
Apple group



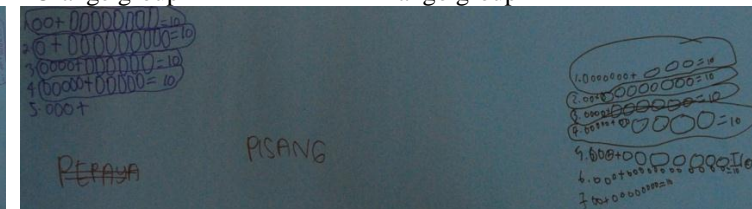
Orange group



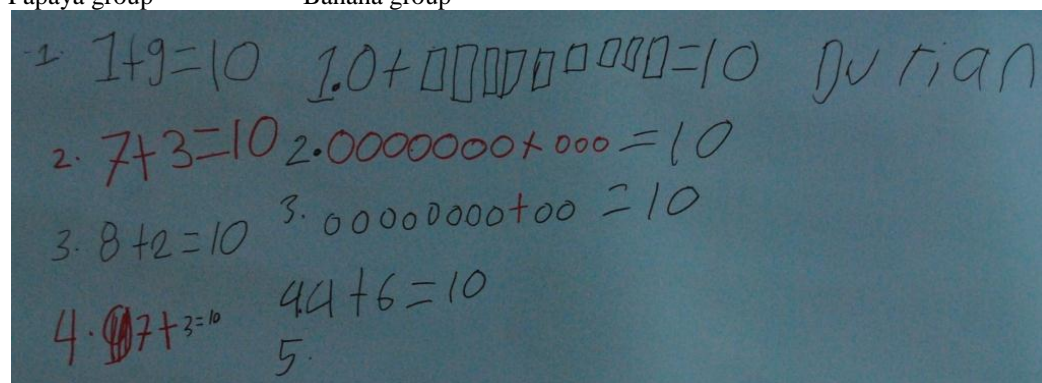
Mango group



Papaya group



Banana group



Durian group

Figure 1: Combinations make ten produced by 6 groups of students.

Episode 3: The teacher and the students actually did not have many experiences in doing a classroom discussion, so we found that the interaction among the students did not appear, and the students were just able to present their tasks and got comments from the teacher. Dimas from the apple group presented their drawing, and he was able to explain the unstructured

drawing and connect the left side as *pempek doses* to the right side as *pempek lenjers* (figure 1). The word “nothing” came from Dimas when he explained there was no *pempek lenjer* when there were 10 *pempek doses*. This process could institutionalize the knowledge about zero when the teacher gave a room for more classroom discussion.

On the other hand, Rizki from the durian group was not able to explain the drawing does by his group, so that the process of *vertical mathematization* is really hard to moving to others students. It seems that the *didactic contract* failed to be broken by the students in the classroom discussion. The role of the teachers was really needed to bridge the gap among students, but we found that she was not able to put herself in the correct way since the lack of experiences in the classroom discussion.

Episode 4: Based on the *intertwinement principle* of RME, the lesson continued the activity using tablets of medicine. First, the teacher showed 10 tablets of medicine to the students and asked them how many they were. In this case, the students could recognize the number of tablets mentally. When she asked how they knew that there were 10 tablets, their argument was not always based on the structure of the tablets of medicine such as 5 and 5, but also based on the combinations that make ten, for instance 4 and 6. Here, the *milieu* emerging in this research was not really useful for students learning. However, when she showed 9 tablets of medicine, they reason based on the tablet structures.

Teacher : How many medicines are these? (The teacher shows 9 tablets of medicine)

All students raise their hands.

Teacher : Nailanda.

Nailanda : 9.

Teacher : How do you know these are 9?

Nailanda : Counted by heart.

Teacher : Explain it!

Nailanda : (Then he shows by his fingers and says weakly five and five minus 1) subtracted by 1.

Teacher : So Nailanda said 5 on the top, 5 on the bottom, and subtracted by 1, so that is 9, isn't it?

Nailanda : Yes.

Teacher : Do you have other ideas? Sahira?

Sahira : 10 minus 1.

The use of 9 tablets of medicine provided the process of institutionalized knowledge from addition to subtraction (*meso-contract*). Nailanda, for instance, was able to see the relations among certain numbers, $5+5-1=9$. Meanwhile, Sahira showed that those were $10 - 1 = 9$. Here, we said that both students provided a *micro-contract of individual production*, and the teacher gave the students an opportunity to appropriate new knowledge, and she institutionalized it. Then, the teacher asked Sena, and he gave an answer as follows:

Sena : 9 minus 1!

Teacher : is that true?

Students : Wrong.

Teacher : Please come here.

When Sena, gave a wrong answer, 9 minus 1, the teacher asked other students to validate this answer. She also asked him to show his answer in front of class. He said that there were 9 tablets of medicine, and one tablet was empty, so it was 9 minus 1. However, other students did not agree with his explanation and said that the result of 9 minus 1 was 8 instead of 9. They validated the wrong interpretation of Sena's answer about the way to know the number of medicine.

DISCUSSION

The use of a real life contextual situation based on *reality principle* as a *milieu* in TDS gives a chance for some students to institutionalize their knowledge. However, other students are not influenced by this situation, but they prefer to use fingers or mental arithmetic to find combinations that make ten. It can be seen as a phenomenon for a certain teaching and learning activities especially when the teacher or researchers try to use and guide students based on their design. We should be aware that the teacher cannot be confident that the students will learn exactly what the teacher intends to teach (Sarrazy & Novotná, 2013). The students probably have their own strategies or models to know a certain mathematical knowledge based on their experiences or old knowledge.

The character from this research is that the teacher has an important role to help students to the process of institutionalized knowledge. It is collateral with the core principle of RME, the *guidance principle*. However, the lack of her experiences in the classroom discussion becomes obstacles for transforming mathematical knowledge among students. So, the process of *vertical mathematization* or *adidactic contract* occurring by a group of students do not influence others, but we cannot fully blame the teacher since the students themselves never have some experiences in carry on their thought in the classroom discussion. We can say that it is a common situation in a traditional teaching and learning practices and need sometimes to change it into an interactive classroom practice.

On the other hand, when the teacher provokes questions in order to validate students' answers, the students are able to communicate their ideas. In this process, they are able to break the contracts in the level of *micro-contract* of agreement. They are also really enthusiastic to engage in the process of individual production and collective production (the level of *micro-contract*). So, the implementation of the *interactivity principle* contributes to break the didactic contracts.

CONCLUSIONS

As a conclusion, the characteristic features of the *didactic contracts* such as *formulation and validation*, occur in the realistic mathematics education teaching and learning practice in the case of learning combinations that make ten. Those features really relate to RME core principles, but there are some considerations that we should aware when we design a classroom teaching and learning based on a certain theory that can make the process of learning by students do not occur naturally. In this study, we found that the teacher gets difficulties to institutionalize new knowledge that is out of the teaching and learning design founded by some students. It makes that some *didactic contracts* fail to be break in the setting of the RME teaching and learning practice.

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